REMARKS

In the January 10, 2008 Office Action, claim 1 was objected to for "insufficient antecedent basis." In addition, claims 1, 3-6, 8-11, and 17-21 were rejected under 35 U.S.C. § 102(b) as being anticipated by U.S. Patent No. 5,796,842 to Hanna ("Hanna"); and claims 2, 7, and 12-16 were rejected under 35 U.S.C. § 103(a) as being unpatentable over Hanna. In response, Applicants have amended the claims to overcome the antecedent basis objection, but have also clarified selected aspects of the claims to remove redundancies or extraneous limitations so that not every amendment is a narrowing amendment submitted in response to a statutory requirement for patentability. Accordingly, Applicants respectfully traverse the rejections for the reasons set forth hereinbelow.

A. Claim 1 Has Been Amended to Remove Inadvertently Included Language

In response to the Examiner's objection to claim 1, Applicants have amended the claim to remove the "input low pass" language which was inadvertently included in the claim. With the amendment, claim 1 now conforms to the remainder of the claims, and now has proper antecedent basis. Accordingly, Applicants request that the objection to claim 1 be withdrawn and that the claim be allowed.

B. Claims 1, 3-6, 8-11 and 17-21 Are Not Anticipated by Hanna

In response to the Examiner's rejection of claims 1, 3-6, 8-11, and 17-21 as being anticipated by Hanna, Applicants respectfully request reconsideration and withdrawal of the rejection because Hanna does not teach, disclose or otherwise make obvious the invention as presently claimed. As a preliminary matter, Applicants would note that the Hanna reference was explicitly referenced in the "Background of Invention" section as typifying the prior art approach over which the present invention expressly provides one or more improvements. *See*, Application, paragraph 12 (pages 5-6). In particular, Hanna's disclosed digital BTSC encoder, which uses conventional low pass filter sections at a relatively low sampling frequency, deviates from the theoretical ideal specified by the BTSC standard, and as a consequence must include compensation devices which deliberately introduce a compensating phase or magnitude error in the encoding process. *See, e.g.*, Hanna Patent, col. 7, lines 26-34 ("Yet another object of the present invention is to provide a digital BTSC encoder including a sum channel processing section for generating the conditioned sum signal, and a difference processing section for generating the encoded difference signal, <u>the sum channel processing section including devices</u>

for introducing compensatory phase errors into the conditioned sum signal to compensate for any phase errors introduced into the encoded difference signal by the difference channel processing section.") (emphasis added). In contrast, Applicants have disclosed and claimed a BTSC encoder which uses higher order IIR digital filters implemented with an allpass decomposition architecture and operated at a sufficiently high sample rate to substantially match BTSC analog filter transform functions in both magnitude and phase. *See*, Application, page 8 (paragraph 16) ("In accordance with the present invention, an integrated circuit system and method are provided for digitally encoding stereophonic audio signals in accordance with the BTSC standard. In a selected embodiment, an improved digital difference channel processing section is provided with an infinite impulse response (IIR) filter, such as higher order elliptical filter, which is implemented using an allpass decomposition filter structure that operates at a high sampling rate.").

1. Hanna Does Not Disclose Digital Filters Implemented Using Allpass Decompositions

As a first point of departure, Hanna does not teach, disclose or otherwise meet the claim requirement that the higher order IIR digital filter be implemented using an "allpass decomposition" architecture as variously recited in claims 1-10 and 20-21. On this point, Applicants have amended claim 20 to more clearly capture the allpass decomposition architecture, though Applicants submit that the original claim language effectively recited the same benefits obtained from using lower order allpass IIR filters in such an architecture, namely eliminating limit cycle oscillations and obtaining a flat response. In reciting that the allpass decomposition filter structure includes a sum of two allpass filter stages, where each allpass filter stage comprises a plurality of lower order allpass IIR filters, claim 20 conforms to Applicants' description and depiction of Figure 7. *See*, Application, page 10 (paragraph 26) ("FIG. 7 depicts an eleventh order elliptical Cauer filter implemented using an allpass decomposition.").

In rejecting claims 1, 3-6, 8-11, and 17-21, the Examiner cites Hanna's description of the low pass filters 224, 238 shown in Figures 3 and 4A and described at col. 11, lines 11-23 and 47-49 as disclosing the requirement in claim 1 of "a higher order IIR digital filter implemented using an allpass decomposition architecture." *See*. Office Action, p. 2. However, a careful reading of the cited passage from Hanna reveals only a generic description (Equation 4) of a second order IIR filter section that is sequentially cascaded to form a tenth order IIR filter, and there is absolutely no reference whatsoever to an "allpass decomposition" architecture, which is

understood by those of ordinary skill in the art (and exemplified in Applicants' Figure 7) to refer to the sum of two allpass filter stages to result in a transfer function $H_0(z) = (A_0(z) + A_1(z)) / 2$. *See*, P. P. Vaidyanathanm, Multirate Systems And Filter Banks, 84 (1992) (attached as Exhibit A); K Iga et al., Encyclopedic Handbook of Integrated Optics, pp. 231-232 (2005) (attached as Exhibit B). The failure by Hanna to describe an allpass decomposition filter structure is plainly and irrefutably confirmed in Hanna's Figures 4A and 4B and associated description which shows that the low pass filters 224, 239 are each formed with a sequential cascade of second order IIR filters, and not by summing two allpass filter stages in an allpass decomposition structure. This deficiency in the Hanna disclosure also appears to be confirmed by the listing of filter coefficients in Hanna's Table 1 for the low pass filter sections 310, 312, 314, 316, 318 for low pass filter 224 which do not appear, at least upon preliminary review, to correspond to allpass filter structures, though further analysis may be required for confirmation. At a minimum, this significant difference is sufficient to overcome the rejections of claims 1-10 and 20-21.

2. Hanna's Disclosed Digital Filters Do Not Operate At A Sample Rate To Match BTSC Analog Filter Transform Functions In Both Magnitude And Phase

As a second point of departure, Hanna does not meet the requirement that the higher order digital filter operate at a "sample rate to substantially match BTSC analog filter transform functions in both magnitude and phase" as variously recited in claim 1-19. *See*, claim 1 (wherein the higher order digital filter, matrix means, sum channel processing means and the difference channel processing means operate at a first sample rate to substantially match BTSC analog filter transform functions in both magnitude and phase.") and claim 11 ("wherein the digital BTSC encoder operates at a sample rate of at least approximately 150-200 kHz so that said digital filters in the sum channel processor and the difference channel processor substantially match BTSC analog filter transform functions in both magnitude and phase") (emphasis added).

In rejecting claims 1, 3-6, 8-11, and 17-21, the Examiner cites Hanna col. 19, lines 35-61 as disclosing claim 1's requirement that "the higher order IIR digital filter ... operate at a first sample rate to substantially match BTSC analog filter transform functions in both magnitude and phase." *See*, Office Action, p. 3. However, a careful reading of cited passage from Hanna reveals only a description of how filter coefficients are calculated for the variable emphasis filter 560. While there is a reference to a sampling frequency of "47202 Hz," this plainly fails to meet the claim 11 requirement of a "sample rate of at least approximately 150-200 kHz." Moreover,

the digital filters in Hanna's disclosed BTSC encoder simply do not operate at a sample rate "to substantially match BTSC analog filter transform functions in both magnitude and phase" as variously recited in claims 1-19. This is most readily confirmed by the fact that Hanna deliberately includes compensation devices, such as the static phase equalization filter 228 and the dynamic phase equalization filters 1010, 1012, which deliberately introduce a compensating phase error in the encoding process. *See, e.g.*, Hanna Patent, col. 10, lines 43-49 ("As will be discussed in greater detail below, static phase equalization filter 228 is used to introduce phase errors that compensate for phase errors introduced by difference processing section 230.") and col. 25, lines 47-49 ("Dynamic phase equalization filters 1010, 1012 are used to compensate for phase errors introduced by variable emphasis filter 560 which is used in spectral compression unit 290."). At a minimum, Hanna's failure to disclose a higher order digital filter that operates at a sufficiently high rate to substantially match BTSC analog filter transform functions in both magnitude and phase is sufficient to overcome the rejections of claims 1-19.

There are other deficiencies in the rejection analysis worth noting in passing. For example, Hanna's pre-emphasis filter 222 cited by the Examiner to anticipate claims 8 and 18 (Office Action, pp. 4-5) is a "first order IIR filter" that does not meet the "higher order IIR digital filter" requirement of claims 8 and 18. *See*, Hanna Patent, col. 10, lines 36-39. In addition, Hanna's cascade of filter sections 310, 312, 314, 316, 318 cited by the Examiner to anticipate claims 5 and 6 (Office Action, p. 3) does not meet the "higher order IIR digital filter" requirement of claim 5 which has been amended to more clearly recite that the filter is formed as a sum of multiple cascades of lower order allpass filters. *See*, Hanna Patent, col. 11, lines 11-23. Likewise, Hanna's variable emphasis filter 560 cited by the Examiner to anticipate claims 10 and 19 (Office Action, pp. 4-5) is a "first order IIR filter" (Hanna Patent, col. 15, lines 53-56) that does not meet the "higher order IIR digital filter" requirement of claims 10 and 19, though it is unclear what is being referenced by the citation to the spectral bandpass filter 542 at col. 14, lines 51-55.

For at least the foregoing reasons, Applicants respectfully request reconsideration and withdrawal of the rejection because the Examiner has not made the *prima facie* anticipation showing that each and every element of the claimed invention, arranged as required by the claims, are found in the Hanna reference, either expressly or under the principles of inherency. *See generally*, In re King, 801 F.2d 1324, 1326, 231 USPQ 136, 138 (Fed. Cir. 1986);

Lindemann Maschinenfabrik GMBH v. American Hoist and Derrick, 730 F.2d 1452, 1458, 221 USPQ 481, 485 (Fed. Cir. 1984). For at least the foregoing reasons, Applicants respectfully submit that the anticipation rejection of claims 1, 3-6, 8-11, and 17-21 be withdrawn and that the claims be allowed.

C. Claims 2, 7, and 12-16 Are Not Obvious

In response to the Examiner's various obviousness rejections of claims 2, 7, and 12-16, Applicants respectfully request reconsideration and withdrawal of these rejections because the Examiner has not established a *prima facie* case of obviousness by showing that all the claim limitations are taught or suggested by the prior art. In re Royka, 490 F.2d 981, 180 USPQ 580 (CCPA 1974); In re Wilson, 424 F.2d 1382, 1385, 165 USPQ 494, 496 (CCPA 1970). In particular, the Examiner relies heavily on "Official Notice" to meet various admittedly missing requirements in claims 2, 7, 12-14 and 16 (e.g., regarding Cauer and Butterworth filters), but in none of the statements of "Official Notice" is there any reference to the higher order digital filter having an "allpass decomposition architecture" or meeting the "magnitude and phase" matching requirements of the claims. Applicants trust that there will not be a revised statement of "Official Notice" to remedy this deficiency since that would appear to be engaging in improper hindsight. If the Examiner takes "Official Notice" that the "allpass decomposition architecture" and "magnitude and phase" matching requirements of claims 2, 7, 12-14 and 16 would be well known, Applicants hereby challenge this assertion pursuant to MPEP § 2144.03(C), and request that the Examiner provide documentary evidence to support this assertion.

Upon determining that claims 12 and 13 contain largely overlapping subject matter, Applicants have amended claim 13 to recite that the higher order IIR digital filter comprises a sum of multiple cascades of lower order allpass filters. Accordingly, and for the reasons set forth above, Applicants submit that claim 13 is allowable over the Hanna reference.

As for the rejection analysis of claim 15, Applicants respectfully submit that the cited combination of Hanna's static phase allpass filter 228 and the multi-section low pass filter 224 does not meet the requirement of a "plurality of second order allpass filters" in claim 15, since the cited Hanna passage (col. 11, lines 10-23) describes each low pass filter section with reference to Equation 4, which is a generic description of a second order low pass filter, particularly when taken in combination with the "low pass filter" coefficients shown in Hanna Table 1 for filter sections 310, 312, 314, 316, 318. Accordingly, Applicants respectfully request

that the obviousness rejection of claims 2, 7, and 12-16 be withdrawn and that the claims be allowed.

CONCLUSION

In view of the remarks set forth herein, Applicants respectfully submit that all pending claims are in condition for allowance and request that a Notice of Allowance be issued. Nonetheless, should any issues remain that might be subject to resolution through a telephone interview, the Examiner is requested to telephone the undersigned at 512-338-9100.

FILED ELECTRONICALLY
April 10, 2008

Respectfully submitted,

/Michael Rocco Cannatti/

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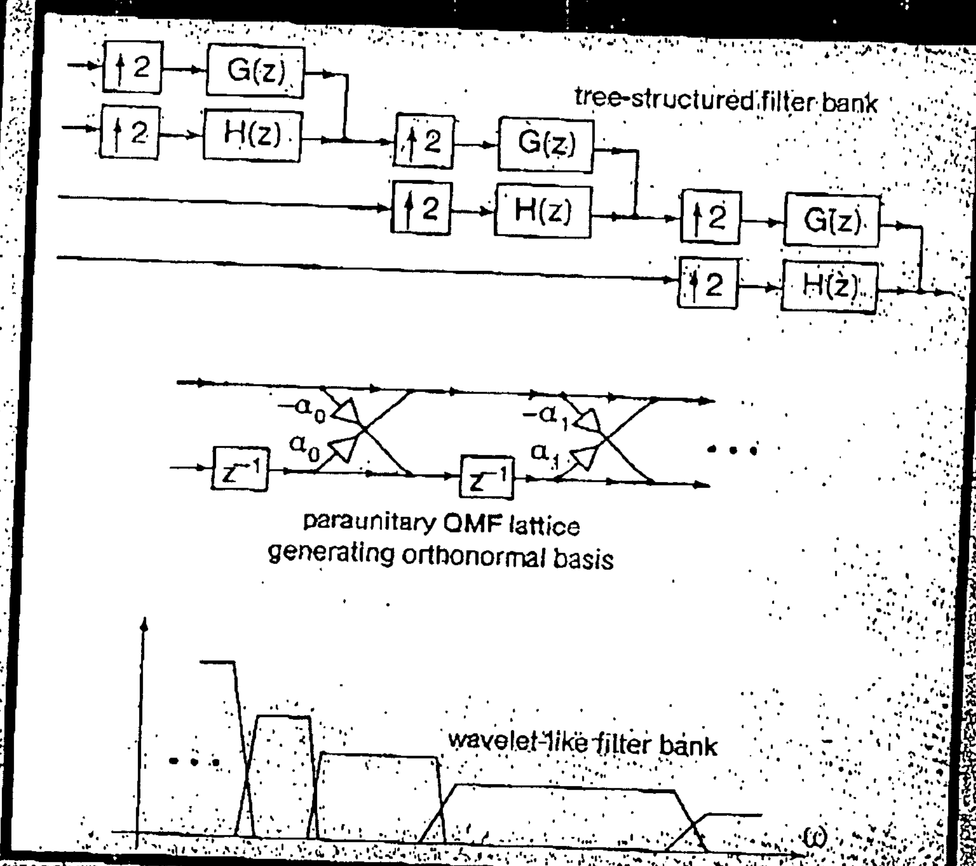
Reg. No. 34,791

Exhibit A - P. P. Vaidyanathanm, <u>Multirate Systems And Filter Banks</u> (1992)

P.P. VAIDYANATHAN

MULTIRATE SYSTEMS AND FILTER BANKS



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with $z = e^{j\omega}$. Since our filters are always rational functions, this condition holds *for all* z (Sec. 2.4.3). In practice the transfer functions are scaled so that $c^2 = 1$. Thus, if $H_0(z)$ is a good lowpass filter, then $H_1(z)$ is a good highpass filter. As a generalization, a set of M transfer functions $H_k(z)$ is said to be power complementary if

$$\sum_{k=0}^{M-1} |H_k(e^{j\omega})|^2 = c^2 > 0, \quad \text{for all } \omega. \tag{3.5.2}$$

This concept will be used in many chapters.

7. *Mth band or Nyquist(M) filters.* These will be described in Section 4.6.1.

3.6 IIR FILTERS BASED ON TWO ALLPASS FILTERS

3.6.1 The Allpass Decomposition Theorem

A wide family of practical transfer functions including Butterworth, Chebyshev, and elliptic filters can be represented as

$$H_0(z) = \frac{A_0(z) + A_1(z)}{2}$$

where $A_0(z)$ and $A_1(z)$ are stable unit-magnitude allpass filters. This has been observed by a number of authors, for example, Fettweis [1974], Constantinides and Valenzuela [1982], Ansari and Liu [1985], Saramäki [1985], and Vaidyanathan, et al. [1986].

The following special case is particularly noteworthy: Let the transfer function $H_0(z)$ be Butterworth, Chebyshev or elliptic *lowpass*, with order N. Let n_0 and n_1 denote the orders of $A_0(z)$ and $A_1(z)$. Then the following things are true.

1. If N is odd, $A_0(z)$ and $A_1(z)$ have real coefficients, and $N = n_0 + n_1$.
2. If N is even, $A_0(z)$ and $A_1(z)$ have complex coefficients and $n_0 = n_1 = N/2$. In this case, the coefficients of $A_1(z)$ are conjugates of those of $A_0(z)$.

The proof of the first statement (odd N) follows from the theorem to be proved next. In this text, only odd N will be of interest, and will be used in Sec. 5.3 (alias-free IIR QMF banks). See Vaidyanathan et al. [1987] for details of even N, which will not be considered here. Also see Problem 3.20.

The fact that a sum of two allpass filters can give rise to good lowpass behavior might occassion an initial surprise. To appreciate the basic idea, recall that the allpass functions have frequency responses $A_0(e^{j\omega}) = e^{j\phi_0(\omega)}$ and $A_1(e^{j\omega}) = e^{j\phi_1(\omega)}$. Now the behavior of the magnitude of

$$H_0(e^{j\omega}) = \frac{e^{j\phi_0(\omega)} + e^{j\phi_1(\omega)}}{2}. \tag{3.6.1}$$

is governed by the phase difference $\phi_0(\omega) - \phi_1(\omega)$. From Sec. 3.4.1 we know that the phase responses of stable allpass filters are monotone decreasing functions. Figure 3.6-1 shows typical sketches of $\phi_0(\omega)$ and $\phi_1(\omega)$ which will ensure that $H_0(z)$ is a good lowpass filter. In the passband $\phi_0(\omega) \approx \phi_1(\omega)$ so that $|H_0(e^{j\omega})| \approx 1$. In the stopband $\phi_0(\omega) - \phi_1(\omega) \approx \pi$ so that $|H_0(e^{j\omega})| \approx 0$.

Thus, an appropriate behavior of relative phases of the two allpass filters can give rise to a good lowpass response. More generally, we will now state and prove the following result.

♠**Theorem 3.6.1. Allpass decomposition.** Let $H_0(z)$ and $H_1(z)$ be two Nth order bounded real (BR) transfer functions (Sec. 3.5) with irreducible rational forms $H_0(z) = P_0(z)/D(z)$ and $H_1(z) = P_1(z)/D(z)$ where,

$$P_0(z) = \sum_{n=0}^{N} p_{0,n} z^{-n}, \quad P_1(z) = \sum_{n=0}^{N} p_{1,n} z^{-n}, \quad D(z) = \sum_{n=0}^{N} d_n z^{-n}. \qquad (3.6.2)$$

Suppose the following conditions are satisfied:

1. $P_0(z)$ is symmetric and $P_1(z)$ antisymmetric, that is,

$$p_{0,n} = p_{0,N-n}, \quad p_{1,n} = -p_{1,N-n}. \qquad (3.6.3)$$

2. $H_0(z)$ and $H_1(z)$ are power complementary, satisfying (3.5.1b) with $c = 1$.

Then $H_0(z)$ and $H_1(z)$ can be expressed as

$$H_0(z) = \frac{A_0(z) + A_1(z)}{2}, \qquad (3.6.4)$$

$$H_1(z) = \frac{A_0(z) - A_1(z)}{2}, \qquad (3.6.5)$$

where $A_0(z)$ and $A_1(z)$ are stable real coefficient allpass functions

$$A_0(z) = \frac{z^{-n_0}\widetilde{D}_0(z)}{D_0(z)}, \quad A_1(z) = \frac{z^{-n_1}\widetilde{D}_1(z)}{D_1(z)}, \qquad (3.6.6)$$

with orders n_0 and n_1, respectively. Moreover $N = n_0 + n_1$. ◇

Comments

1. The BR nature of $H_0(z)$ and $H_1(z)$ means that these are stable, that the coefficients $p_{0,n}, p_{1,n}$ and d_n are real and that the magnitudes are bounded by unity.
2. The allpass functions $A_0(z)$ and $A_1(z)$ have unit magnitude and their orders $A_0(z)$ and $A_1(z)$ add up to N.

3. Out of the N poles of $H_0(z)$, a subset of n_0 poles are assigned to $A_0(z)$ and the remaining n_1 poles assigned to $A_1(z)$. This partitioning of the poles of $H_0(z)$ *completely* determines its numerator. The zeros of $H_0(z)$ are, therefore, *not* independent parameters any more. The transfer function has only N degrees of freedom.
4. Figure 3.6-2 indicates a structure which implements the two transfer functions.

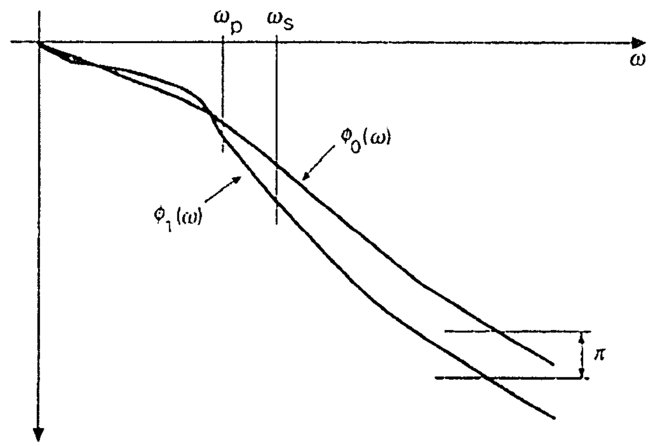


Figure 3.6-1 Demonstrating the phase responses of the two allpass functions.

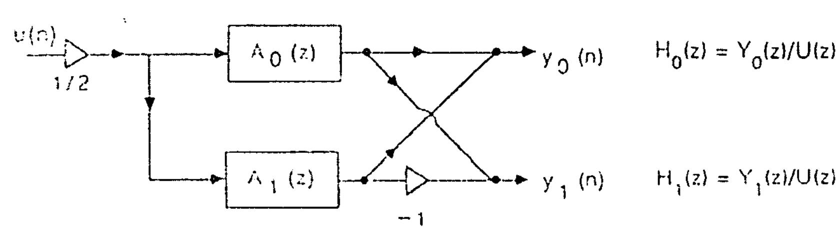


Figure 3.6-2 Implementing two transfer functions by adding and subtracting two allpass filters.

Proof of Theorem 3.6.1. First notice that (3.5.1b) can be rearranged as

$$\widetilde{P}_0(z)P_0(z) + \widetilde{P}_1(z)P_1(z) = \widetilde{D}(z)D(z), \tag{3.6.7}$$

since $c^2 = 1$. In view of (3.6.3) we have

$$\widetilde{P}_0(z) = z^N P_0(z), \quad \widetilde{P}_1(z) = -z^N P_1(z). \tag{3.6.8}$$

Substituting into (3.6.7) we obtain $P_0^2(z) - P_1^2(z) = z^{-N}\widetilde{D}(z)D(z)$, which can be rewritten as

$$\Big(P_0(z) + P_1(z)\Big)\Big(P_0(z) - P_1(z)\Big) = z^{-N}\widetilde{D}(z)D(z). \tag{3.6.9}$$

Notice that $P_0(z) - P_1(z) = z^{-N}\left(\tilde{P}_0(z) + \tilde{P}_1(z)\right)$ so that the zeros of $P_0(z) - P_1(z)$ are the reciprocal conjugates of those of $P_0(z) + P_1(z)$.

We know that the zeros of $D(z)$ are inside the unit circle so that those of $\tilde{D}(z)$ are outside. So, none of the zeros of $P_0(z) + P_1(z)$ can be on the unit circle (from (3.6.9)). Let n_1 be the number of zeros of $P_0(z) + P_1(z)$ inside the unit circle. Then, there is a factor of $D(z)$, denote it $D_1(z)$, of order n_1 which is also a factor of $P_0(z) + P_1(z)$. Clearly, $P_0(z) + P_1(z)$ has $n_0 \triangleq N - n_1$ zeros outside the unit circle. As seen from (3.6.9) there is then a factor of $\tilde{D}(z)$, say $\tilde{D}_0(z)$, of order n_0 which is also a factor of $P_0(z) + P_1(z)$. Clearly $D_0(z)$ is a n_0th order factor of $D(z)$. Summarizing we can always write

$$P_0(z) + P_1(z) = \alpha D_1(z) z^{-n_0} \tilde{D}_0(z), \tag{3.6.10}$$

where

$$D_0(z) = 1 + \sum_{n=1}^{n_0} d_{0,n} z^{-n}, \quad \text{and} \quad D_1(z) = 1 + \sum_{n=1}^{n_1} d_{1,n} z^{-n}, \tag{3.6.11}$$

are factors of $D(z)$, and α is a real nonzero constant.

As the orders of $D_0(z)$ and $D_1(z)$ add up to the order of $D(z)$, we get

$$D(z) = D_0(z) D_1(z). \tag{3.6.12}$$

By using (3.6.10) and (3.6.12) in (3.6.9) we obtain

$$P_0(z) - P_1(z) = \frac{1}{\alpha} D_0(z) z^{-n_1} \tilde{D}_1(z). \tag{3.6.13}$$

But the symmetry relation (3.6.8) along with (3.6.10) also leads to this equation, with α in place of $1/\alpha$. This implies $\alpha = \pm 1$. We take $\alpha = 1$ (because the other choice $\alpha = -1$ does not change the magnitude responses of $H_0(z)$ and $H_1(z)$ anyway). Dividing both sides of (3.6.10) and (3.6.13) by $D(z)$ we finally arrive at

$$H_0(z) + H_1(z) = A_0(z), \quad H_0(z) - H_1(z) = A_1(z). \tag{3.6.14}$$

Rearranging (3.6.14), we therefore obtain (3.6.4) and (3.6.5). $\nabla\nabla\nabla$

3.6.2 Elliptic, Butterworth, and Chebyshev Filters

Figure 3.6-3 shows the typical magnitude response of a fifth-order elliptic lowpass filter $H_0(z) = P_0(z)/D(z)$. The coefficients are known to be real, and the magnitude is bounded by unity so that $H_0(z)$ is BR. We know that all the zeros are on the unit circle. The zero at $\omega = \pi$ contributes to the factor $(1+z^{-1})$ and the complex conjugate pairs of zeros contribute to factors

of the form $(1 - 2z^{-1}\cos\omega_k + z^{-2})$. So, the numerator $P_0(z)$ is indeed a symmetric polynomial.

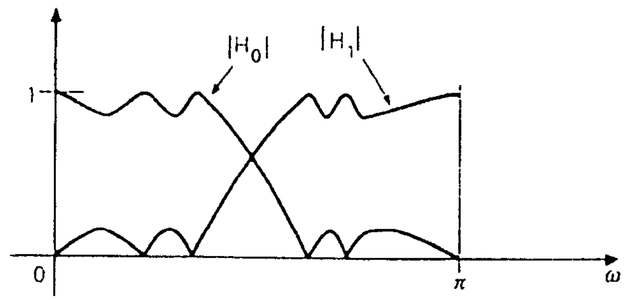


Figure 3.6-3 A fifth order elliptic lowpass filter, and its power complementary response.

The figure also shows the magnitude of the power complementary filter $H_1(z)$. Clearly, $|H_1(e^{j\omega})|$ is equal to zero at frequencies where, $|H_0(e^{j\omega})|$ takes the maximum value of unity. $|H_1(e^{j\omega})|$ has one zero at $\omega = 0$ and two complex conjugate pairs of zeros on the unit circle so that all the zeros are on the unit circle again. The zero at $\omega = 0$, however, contributes to an *antisymmetric factor* $(1 - z^{-1})$. As a result, the numerator $P_1(z)$ of $H_1(z)$ is antisymmetric. Summarizing, $H_0(z)$ has a symmetric numerator and $H_1(z)$ has an antisymmetric numerator.

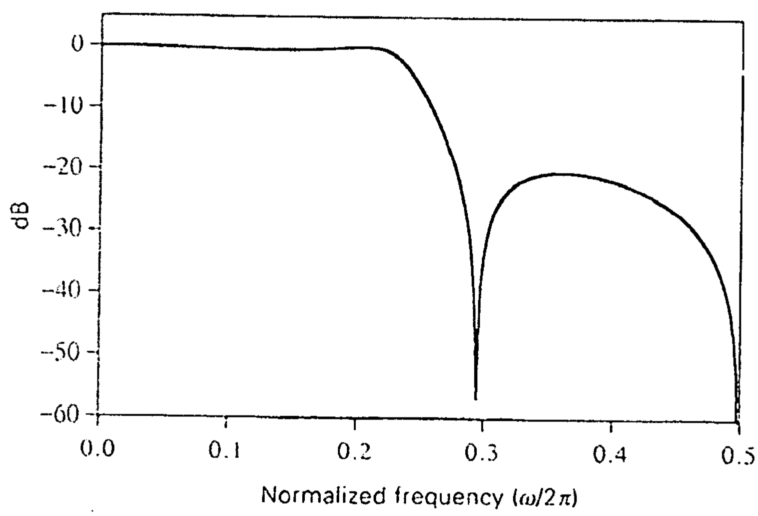


Figure 3.6-4 Example 3.6.1. Magnitude response of a 3rd order elliptic filter.

More generally, if $H_0(z)$ represents an *odd order lowpass* Butterworth, Chebyshev or elliptic filter, the above conclusions remain valid. That is, the

numerator of $H_0(z)$ is symmetric and that of $H_1(z)$ is antisymmetric. We can, therefore, apply Theorem 3.6.1 to conclude that $H_0(z)$ and $H_1(z)$ can be expressed as in (3.6.4) and (3.6.5).

Example 3.6.1

Consider the third order elliptic lowpass filter

$$H_0(z) = \frac{0.23179 + 0.36021z^{-1} + 0.36021z^{-2} + 0.23179z^{-3}}{1 - 0.38409z^{-1} + 0.70390z^{-2} - 0.13581z^{-3}} \tag{3.6.15}$$

whose magnitude response is shown in Fig. 3.6-4. The reader can verify that $H_0(z)$ can be expressed as

$$H_0(z) = 0.5\left[\underbrace{\frac{-0.20356 + z^{-1}}{1 - 0.20356z^{-1}}}_{A_0(z)} + \underbrace{\frac{0.66715 - 0.18053z^{-1} + z^{-2}}{1 - 0.18053z^{-1} + 0.66715z^{-2}}}_{A_1(z)}\right].$$

Evidently $A_0(z)$ and $A_1(z)$ indicated above are unit-magnitude allpass.

Efficiency of the Allpass Based Structure

The cost of the implementation of Fig. 3.6-2 (say an elliptic filter) is equal to the cost of the two allpass filters plus the two adders. We know from Sec. 3.4.3 that a real coefficient allpass filter of order n_k can be implemented with n_k multipliers. So, the structure requires only $n_0 + n_1 = N$ multipliers. For this cost, we get two filters $H_0(z)$ and $H_1(z)$, that is, we require $N/2$ multipliers per filter! In contrast, a direct form implementation of a single elliptic filter would require as many as $1.5N$ multipliers (even after taking numerator symmetry into account)!

The Pole Interlace Property

Given an odd order elliptic transfer function $H_0(z) = P_0(z)/D(z)$, what is the procedure to identify the allpass functions $A_0(z)$ and $A_1(z)$? One method would be to identify $P_1(z)$ using (3.6.7), and compute the zeros of $P_0(z) + P_1(z)$. The zeros inside the unit circle determine $D_1(z)$, and those outside are used to determine $D_0(z)$. The allpass functions can then be found from (3.6.6).

There exists a simpler procedure, whenever the zeros of $D(z)$ [poles of $H_0(z)$] are known. Let the poles of $H_0(z)$ be $z_0, z_1, \ldots$, with pole angles $\theta_0, \theta_1, \ldots$. Let the numbering of poles be such that $\theta_0 < \theta_1 < \ldots$. Then the poles of $A_0(z)$ are given by z_{2k} and those of $A_1(z)$ by z_{2k+1}. This is called the *pole interlace property* [Gazsi, 1985]. Using this we can identify the allpass functions as demonstrated in Fig. 3.6-5.

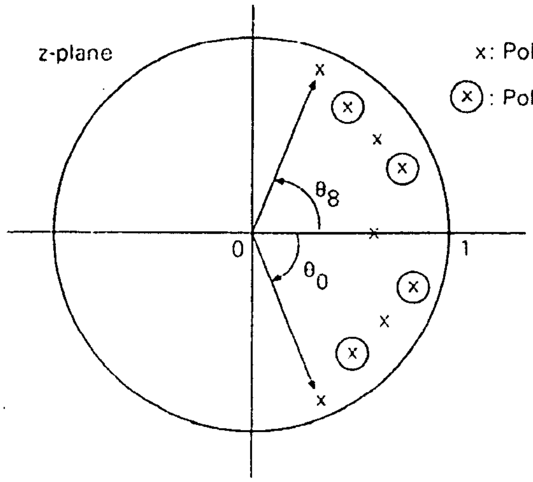


Figure 3.6-5 Demonstration of interlace property. The nine poles of $H_0(z)$ are split into those of $A_0(z)$ and $A_1(z)$ as indicated.

Case When N is Even

What happens if the filter has even order? Consider a sixth order elliptic lowpass filter $H_0(z) = P_0(z)/D(z)$ with response as shown in Fig. 3.6-6. In the region $0 \le \omega \le \pi$, there are three zeros. Thus we have three complex conjugate pairs of zeros, giving rise to three factors of the form $(1 - 2z^{-1}\cos\omega_k + z^{-2})$ for the numerator $P_0(z)$. This numerator, therefore, is symmetric.

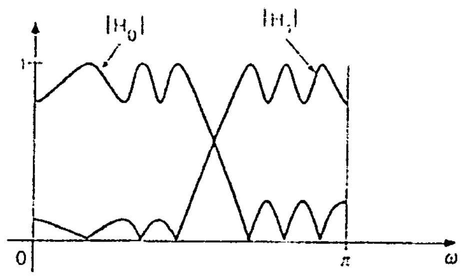


Figure 3.6-6 A sixth order elliptic lowpass filter and its power complementary response.

Now consider the power complementary response $|H_1(e^{j\omega})|$ which is also shown in the figure. This is zero whenever $|H_0(e^{j\omega})|$ is unity. Since $|H_0(e^{j\omega})|$ does *not* have a maximum at $\omega = 0$, we conclude that $|H_1(e^{j\omega})| \neq 0$ at $\omega = 0$. So the numerator $P_1(z)$ of $H_1(z)$ does not have the factor $(1 - z^{-1})$. In fact $P_1(z)$ has three factors of the form $(1 - 2z^{-1}\cos\theta_k + z^{-2})$ because it also has three pairs of complex conjugate zeros on the unit circle. As a result $P_1(z)$ is symmetric rather than antisymmetric. More generally whenever $H_0(z)$ is

a Butterworth, Chebyshev or elliptic lowpass filter of even order, the above conclusion remains true. That is, the numerators of $H_0(z)$ and $H_1(z)$ are both symmetric. So, the conditions of Theorem 3.6.1 are not satisfied.

In this case it can be shown [Vaidyanathan, et. al., 1987] that we can still express $H_0(z)$ as $0.5[A_0(z)+A_1(z)]$, where $A_0(z)$ and $A_1(z)$ are complex-coefficient allpass filters, and the coefficients of $A_1(z)$ are conjugates of those of $A_0(z)$. Finally, note that if $H_0(z)$ is bandpass or bandstop, then it can often be implemented in terms of real allpass filters, even if its order is even. (Example: start from an odd order elliptic lowpass filter and replace z with z^2 or $-z^2$.)

TABLE 3.7.1 Comparison of four techniques for lowpass filter design. The specifications are $\omega_p = 0.15\pi$, $\omega_S = 0.20\pi$, $\delta_1 = 0.01$ and $\delta_2 = 0.001$.

Method	IIR elliptic	IIR Butterworth	FIR equiripple	FIR Kaiser window
Special features	Optimal in minimax sense	Maximally flat at $\omega = 0, \pi$	Linear phase. Also optimal in minimax sense	Linear phase. Very easy to design
Required order N	7	28	101	146
Complexity of implementation	11 mul* 14 add (direct form)	28 mul, 56 add (direct form)	51 mul, 101 add	74 mul, 146 add

* 7 mul and 22 add, if the allpass based structure is used (with one-multiplier lattice sections).

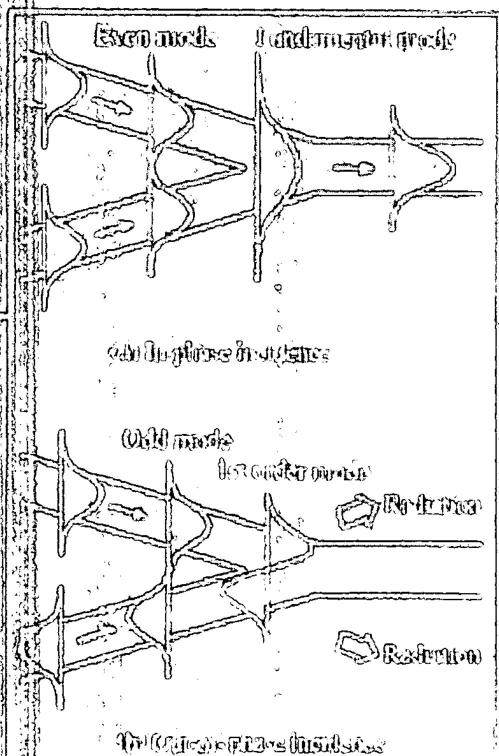
3.7 CONCLUDING REMARKS

In Sec. 3.1 to 3.3 we reviewed many techniques for digital filter design. A summary and comparison of many of the earlier methods can be found in Rabiner and Gold [1975]. In Table 3.7.1 we have compared the filter orders and computational complexities of several methods, for a given set of specifications on the magnitude response. It is clear that the IIR elliptic design is the least expensive, but it introduces phase distortion. The FIR filters, on the other hand, have exact linear phase, but are more expensive. As explained at the beginning of Sec. 3.3, the complexity in terms of multiplications and additions is not always a fair measure of comparison. One should take into account the architecture of the implementation and, if possible, use more efficient FIR implementations (e.g., multistage implementations, Sec. 4.4).

The following Chapters will show that in the context of multirate signal

Exhibit B - K Iga et al., Encyclopedic Handbook of Integrated Optics (2005)

Encyclopedic Handbook of INTEGRATED OPTICS



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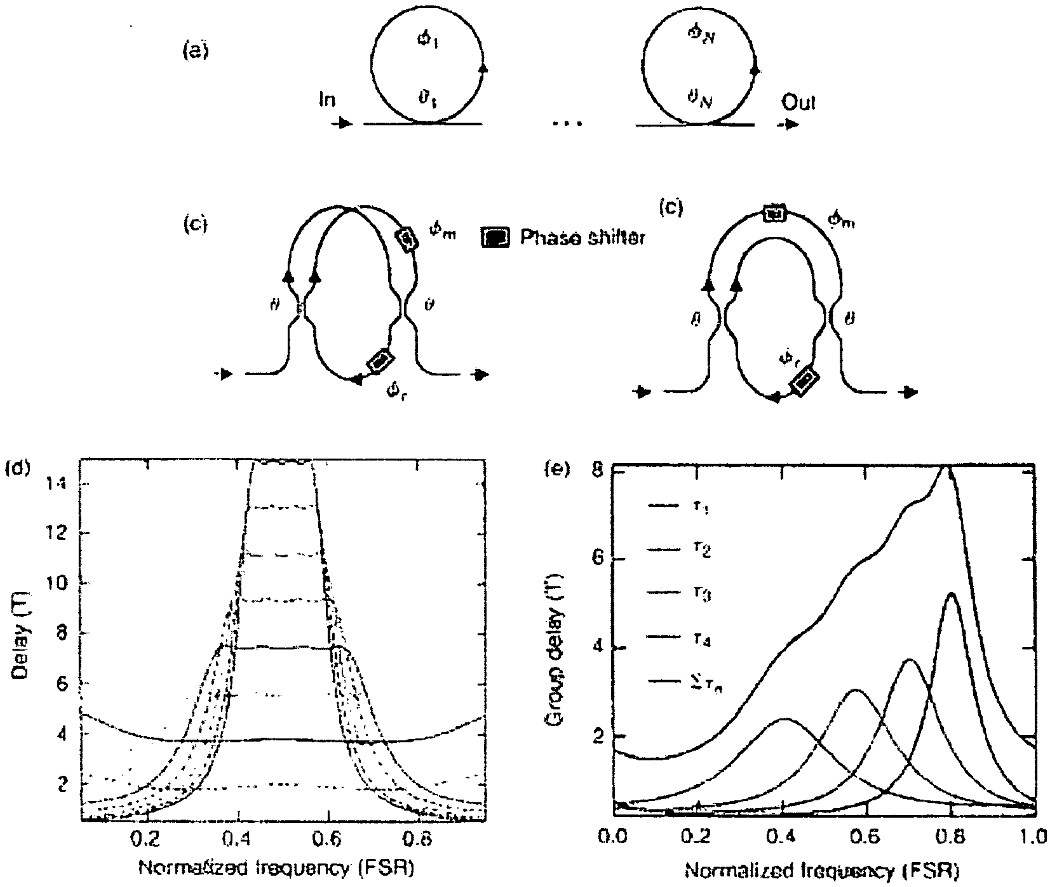

Figure 13 (a) Cascaded multistage allpass filter. Rings incorporating (b) symmetric, and (c) antisymmetric MZIs. Group delay responses for (d) variable delay and (e) tunable dispersion compensating applications

Figure 13(b) and 13(c) to achieve tunable couplers using phase shifters. Since the filtering is intrachannel for dispersion compensators, FSRs equal to the channel spacing are employed that require smaller core-to-cladding index contrasts than interchannel filters requiring FSRs that cover many channels, discussed previously for bandpass filter applications. Tunable dispersion compensators using ring resonators-based allpass filters have been reported in both Ge-doped silica [89] and SiON [90] waveguides. System tests at 10 Gb/sec with a tuning range of 4000 psec/nm [91] and 40 Gb/sec with a tuning range over 200 psec/nm [92] have been performed. More general allpass filter architectures with lattice structures have been proposed [88,93] and demonstrated with etalon implementations [94].

For bandpass filter applications, a steeper transition-band rolloff is achieved if control over both the pole and zero locations is available. A multistage IIR filter architecture with arbitrary pole and zero locations is shown in Figure 14(a) [95]. The stages are coupled making it difficult to precisely set the optical parameters. The simplified architecture in Figure 14(b) places multistage allpass filters within the interferometer arms [96,97]. The resulting transfer functions are sums and differences of allpass filter responses as indicated in Eqs. (20) and (21). This architecture also has many applications in digital filters [98,99].

$$G(z)=\frac{1}{2}\left[A_1(z)+A_2(z)\right]=\frac{P(z)}{D(z)}, \tag{20}$$

$$H(z) = \frac{1}{2}\left[A_1(z) - A_2(z)\right] = \frac{Q(z)}{D(z)}. \tag{21}$$

The two distinct output responses share a common denominator but have different numerator polynomials. The polynomial roots are coupled as a consequence of power conservation, resulting in an equation similar to Eq. (15) for FIR lattice filters [7]. Filter design is accomplished by decomposing the desired response into appropriate allpass filter responses, $A_1(z)$ and $A_2(z)$. Bandpass filters with optimal passband and stopband amplitude characteristics can be directly implemented, such as Butterworth, Chebyshev, and elliptic filter responses [96]. A maximally-flat response was first demonstrated using a ring resonator [100]. A fifth-order elliptic response is shown in Figure 14(c). The allpass decomposition architecture also lends itself to etalon-based implementations [96,101]. In general, IIR bandpass filters require substantially fewer stages to realize a given passband flatness, transition width, and stopband rejection compared with FIR filters as shown in the comparison of Figure 15. The drawback for IIR filters is the dispersion introduced by the poles. An allpass filter may be cascaded with the IIR bandpass

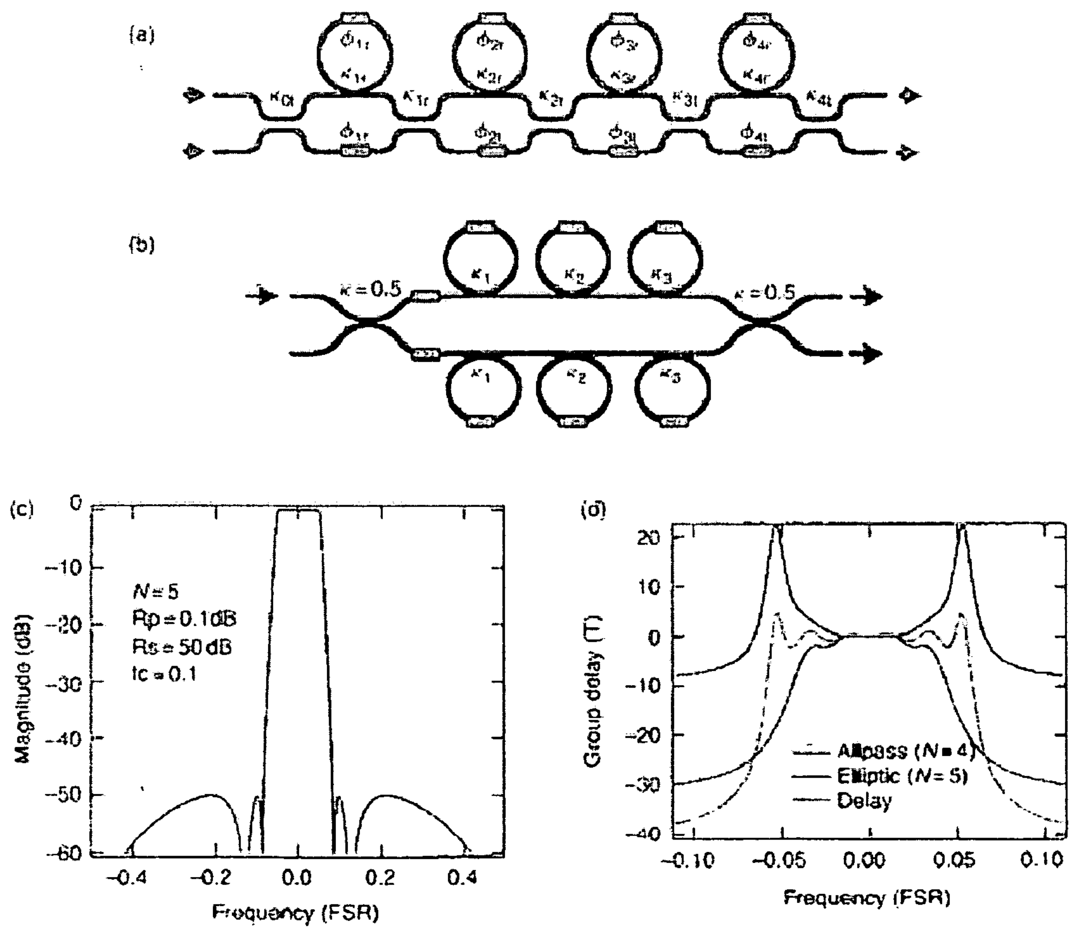

Figure 14 (a) General IIR filter (From K. Jinguji, *J. Lightwave Technol.*, 14, 1882–1898, 1996. With permission. Copyright 1996 IEEE.) and (b) architecture using allpass filter decomposition (From C. Madsen, *IEEE Photonics Technol. Lett.*, 10, 1136–1138, 1998. Copyright 1998 IEEE.) A fifth-order elliptic filter bandpass (c) magnitude and (d) delay response with compensating allpass response (From C. Madesen and G. Lenz, *IEEE Photonics Technol. Lett.*, 10, 994–996, 1998. Copyright 1998 IEEE.)

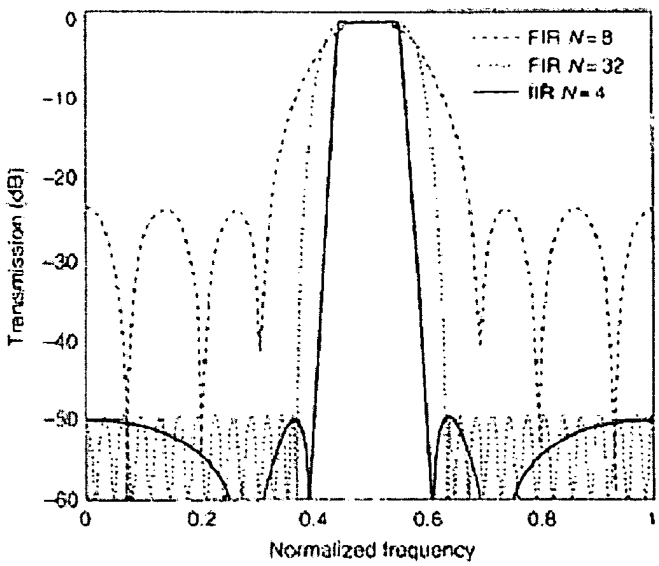

Figure 15 Comparison of bandpass FIR ($N = 8$ and $N = 32$) and IIR ($N = 4$) filter designs

filter to mitigate its dispersion as shown in Figure 14(d); however, this increases the total number of stages and complexity of the implementation. For very large FSRs, the dispersion may become negligible given the physical scaling with the unit delay as indicated in Eq. (14). An arbitrary 2×2 unitary filter, that is, two coupled magnitude and phase responses, can also be realized with a cascade of multistage allpass filters interconnected with couplers and has been applied to PMD emulation/compensation [39]. Notch filters may also be realized using allpass filter decomposition [102,103].

SUMMARY

Optical filters are, clearly, important building blocks for signal processing in optical systems. A great deal of progress has been made in the theory, design, and fabrication of adaptive optical filters within the last few years. A broad range of applications has been demonstrated from reconfigurable optical add/drops multiplexers for the bandwidth management of a large number of optical channels to tunable dispersion compensators for high bitrate optical networks. As per channel bitrates and spectral efficiency increase and systems drive toward optical mesh networks instead of point-to-point links, more ideal filters with agile responses and adaptive filters for dispersion and PMD compensation will be needed.

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